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### Semester One Examination, 2020

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**METHODS**

**UNIT 3**

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (6 marks)

A seafood processor buys batches of prawns from their supplier, where is a constant. In any given batch, the probability that a prawn is export quality is , where is a constant and the quality of an individual prawn is independent of other prawns.

The discrete random variable is the number of export quality prawns in a batch and the mean of is and standard deviation of is .

(a) State the name given to the distribution of and determine its parameters and .

(4 marks)

|  |
| --- |
| **Solution** |
| follows a binomial distribution. |
| **Specific behaviours** |
| ✓ names binomial distribution  ü equation for mean and variance (or sd)  ü value of  ü value of |

(b) Determine the probability that more than 50% of prawns in a randomly selected batch are export quality. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ lower bound  ü probability |

Question 10 (8 marks)

The voltage, volts, supplied by a battery hours after timing began is given by

(a) Determine

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(i) the initial voltage. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(ii) the voltage after hours. (1 mark)

(iii) the time taken for the voltage to reach volts. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(b) Show that and state the value of the constant . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct derivative  ü value of |

(c) Determine the rate of change of voltage hours after timing began. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct rate |

(d) Determine the time at which the voltage is decreasing at of its initial rate of decrease.

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates suitable method  ü correct time |

Question 11 (8 marks)

A small body moving in a straight line has displacement cm from the origin at time seconds given by

(a) Use derivatives to justify that the maximum displacement of the body occurs when .

(4 marks)

|  |
| --- |
| **Solution** |
| Hence when has a stationary point.  Since second derivative is negative, the stationary point is a maximum, and so the body has a maximum displacement when . |
| **Specific behaviours** |
| ✓ first derivative  ü indicates stationary point at required time  ü value of second derivative at required time  ü statement that justifies maximum |

(b) Determine the time(s) when the velocity of the body is not changing. (2 marks)

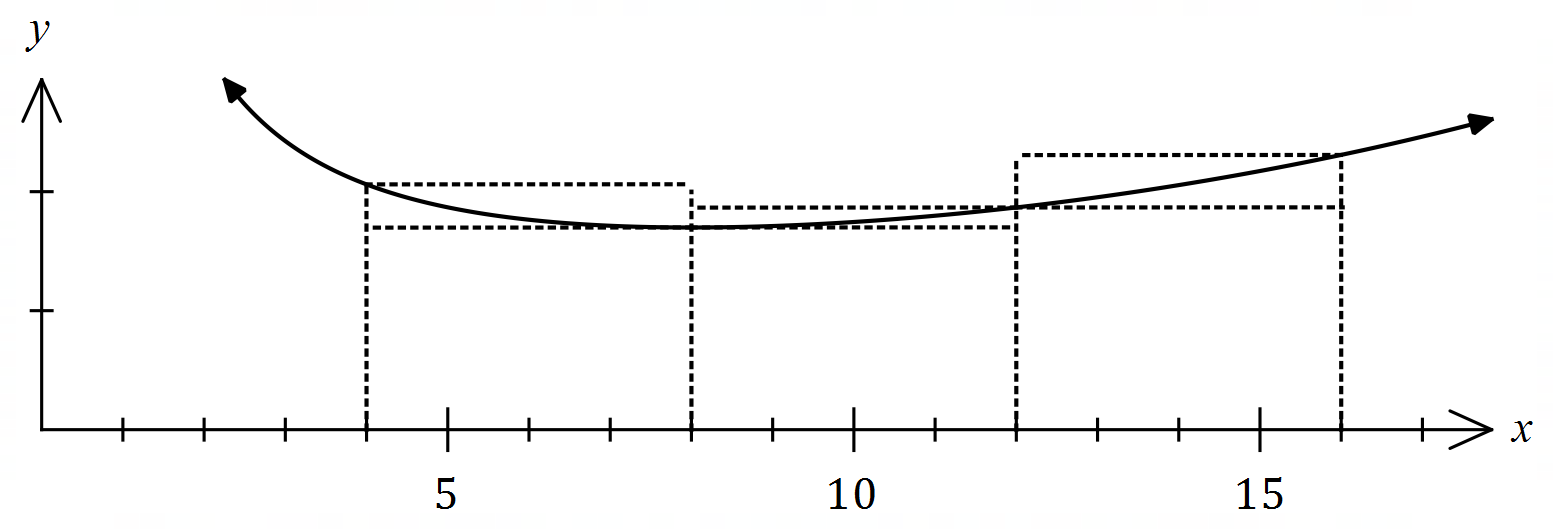
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates acceleration/second derivative must be zero  ü states exact (or approximate) times in interval |

(c) Express the acceleration of the body in terms of its displacement . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ factors out  ü correct expression |

Question 12 (7 marks)

The function is defined as , and the graph of is shown below.



(a) Complete the missing values in the table below, rounding to decimal places. (1 mark)

|  |
| --- |
| **Solution** |
| See table |
| **Specific behaviours** |
| ✓ both correct |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |

(b) Use the areas of the rectangles shown on the graph to determine an under- and over-estimate for . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates  ü under-estimate  ü over-estimate |

(c) Use your answers to part (b) to obtain an estimate for . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct mean |

(d) State whether your estimate in part (c) is too large or too small and suggest a modification to the numerical method employed to obtain a more accurate estimate. (2 marks)

|  |
| --- |
| **Solution** |
| Estimate is too large ( is concave upwards).  Better estimate can be found using a larger number of thinner rectangles. |
| **Specific behaviours** |
| ✓ states too big  ü indicates modification to improve estimate |

Question 13 (8 marks)

A bag contains four similar balls, one coloured red and three coloured green. A game consists of selecting two balls at random, one after the other and with the first replaced before the second is drawn. The random variable is the number of red balls selected in one game.

(a) Complete the probability distribution for below. (3 marks)

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ one correct probability  ü probabilities have sum of   all correct probabilities |

(b) Determine and . (2 marks)

|  |
| --- |
| **Solution** |
| *NB Using CAS,* . |
| **Specific behaviours** |
| ✓ expected value   variance |

(c) A player wins a game if the two balls selected have the same colour. Determine the probability that a player wins no more than three times when they play five games.

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ defines distribution  ü states probability required  ü correct probability |

Question 14 (8 marks)

A curve has equation .

(a) Show that the curve has only one stationary point and use an algebraic method to determine its nature. (3 marks)

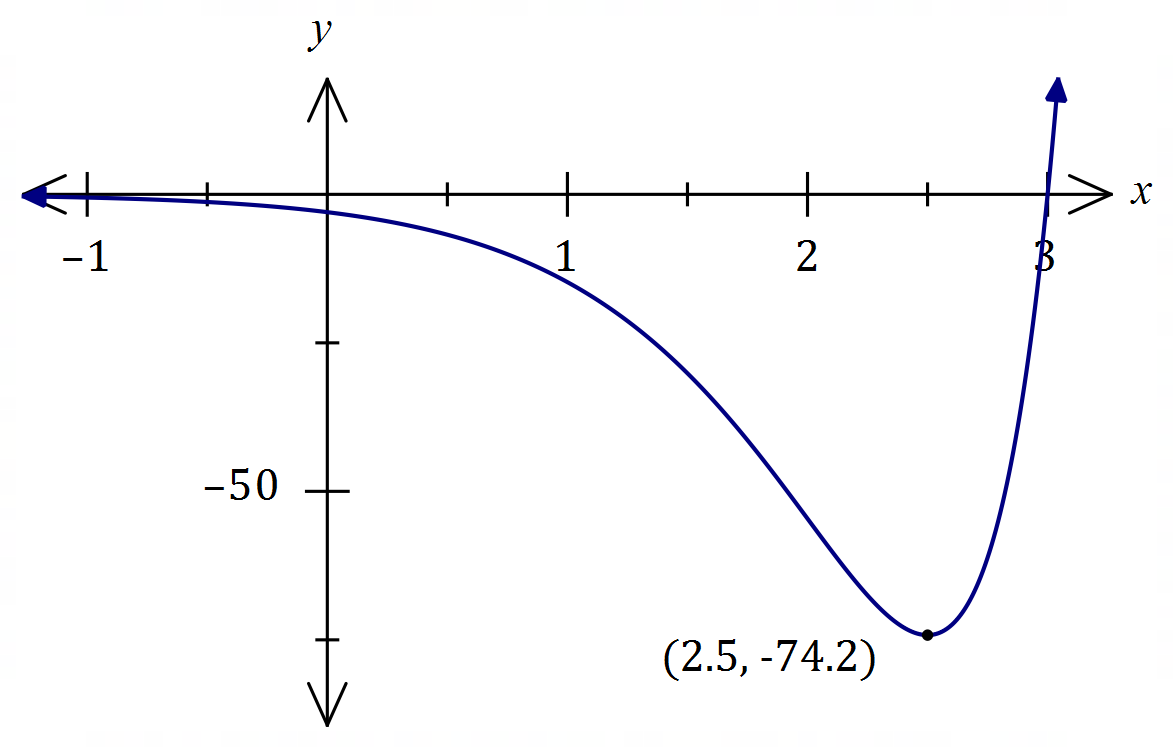
|  |
| --- |
| **Solution** |
| For stationary point, require and since then - there is only one stationary point.  Hence stationary point is a local minimum. |
| **Specific behaviours** |
| ✓ first derivative  ü uses factored form to justify one stationary point  ü indicates minimum using derivatives (sign or 2nd) |

(b) Justify that the curve has a point of inflection when . (3 marks)

|  |
| --- |
| **Alternative Solution** |
| Hence point of inflection as and . |
| **Specific behaviours** |
| ✓ shows second derivative is zero  ü calculates third derivative  ü explains justification |

|  |
| --- |
| **Solution** |
| Hence point of inflection as concavity changes from ve to ve as increases through . |
| **Specific behaviours** |
| ✓ shows second derivative is zero  ü calculates second derivative either side  ü explains justification |

(c) Sketch the curve on the axes below. (2 marks)

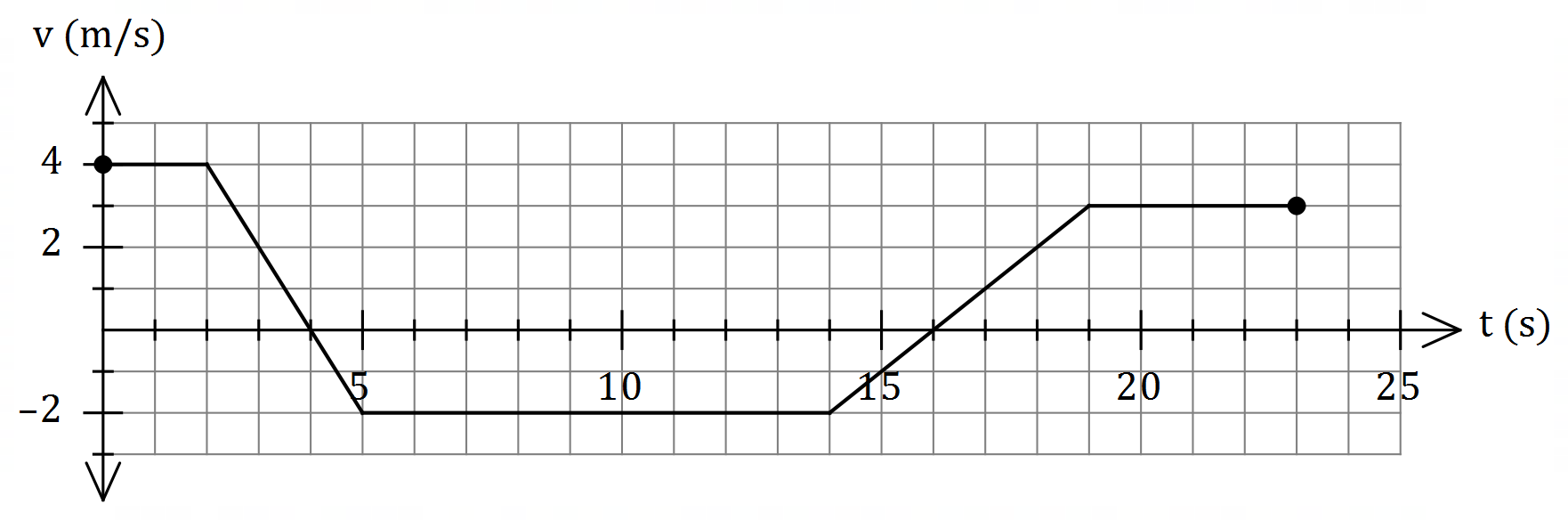


|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ minimum, -intercept  ü correct shape |

Question 15 (9 marks)

A small body leaves point and travels in a straight line for seconds until it reaches point .

The velocity m/s of the body is shown in the graph below for seconds.



(a) Use the graph to evaluate and interpret your answer with reference to the motion of the small body. (3 marks)

|  |
| --- |
| **Solution** |
| The change in displacement of the body during the first seconds is m.  OR  The body has moved m to the right of during first seconds. |
| **Specific behaviours** |
| ✓ value of integral  ü interprets as change in displacement  ü includes specific time and distance with units in interpretation |

(b) Determine an expression, in terms of , for the displacement of the body relative to during the interval . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expression for  ü expression for with constant  ü correct expression for |

(c) Determine the time(s) at which the body was at point for . (3 marks)

|  |
| --- |
| **Solution** |
| Body at point when s and s. |
| **Specific behaviours** |
| ✓ indicates appropriate method using areas  ü one correct time  ü two correct times |

Question 16 (9 marks)

When a machine is serviced, between and of its parts are replaced. Records indicate that of machines need part replaced, need parts replaced, need parts replaced, and the mean number of parts replaced per service is .

Let the random variable be the number of parts that need replacing when a randomly selected machine is serviced.

(a) Complete the probability distribution table for below. (4 marks)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

|  |
| --- |
| **Solution** |
| Let then  Hence |
| **Specific behaviours** |
| ✓ values for  ü equation using sum of probabilities  ü equation using expected value  ü values for |

|  |
| --- |
| **Solution** |
| Using CAS,  Hence |
| **Specific behaviours** |
| ✓ indicates sd using CAS  ü correct variance |

(b) Determine . (2 marks)

The cost of servicing a machine is plus per part replaced and the random variable is the cost of servicing a randomly selected machine.

(c) Determine the mean and standard deviation of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equation relating and  ü mean  ü standard deviation (penalty no units: mark) |

Question 17 (6 marks)

Some values of the polynomial function are shown in the table below:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

(a) Evaluate . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses fundamental theorem  ü correct value |

The following is also known about :

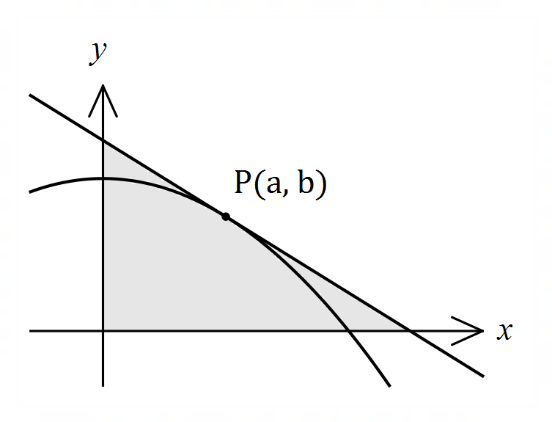
|  |  |  |  |
| --- | --- | --- | --- |
| Interval |  |  |  |
|  |  |  |  |

(b) Determine the area between the curve and the -axis, bounded by and . (4 marks)

|  |
| --- |
| **Solution** |
| Area to left of is above axis but to left is below so will need to negate/drop negative sign for that integral: |
| **Specific behaviours** |
| ü integral for  ü negated integral for  ü uses fundamental theorem  ü correct area |

Question 18 (8 marks)

Let be a point in the first quadrant that  
lies on the curve and be the area  
of the triangle formed by the tangent to the  
curve at and the coordinate axes.



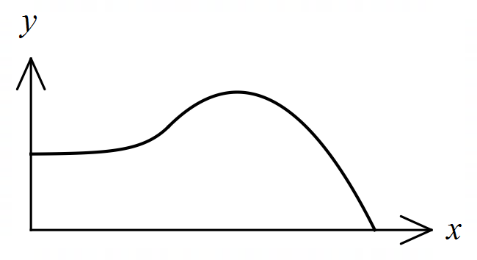
(a) Show that . (4 marks)

|  |
| --- |
| **Solution** |
| Gradient at :  Equation of tangent:  Axes intercepts:  Area: |
| **Specific behaviours** |
| ✓ in terms of and  ü equation of tangent in terms of (any form)  ü axes intercepts  ü indicates area of right triangle |

(b) Use calculus to determine the coordinates of that minimise . (4 marks)

|  |
| --- |
| **Solution** |
| Hence |
| **Specific behaviours** |
| ✓ first derivative  ü solves for  ü indicates check for minimum (graph, sign or second derivative test)  ü correct coordinates, exact or at least dp |

Question 19 (7 marks)

The edges of a swimming pool design, when viewed  
from above, are the -axis, the -axis and the curves

and

where and are measured in metres.

(a) Determine the gradient of the curve at the point where the two curves meet. (2 marks)

|  |
| --- |
| **Solution** |
| Curves intersect when |
| **Specific behaviours** |
| ✓ -coordinate of intersection  ü common gradient |

(b) Determine the surface area of the swimming pool. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ upper bound for parabola  ü area  ü area  ü total area, with units |

(c) Given that the water in the pool has a uniform depth of cm, determine the capacity of the pool in kilolitres ( kilolitre of water occupies a volume of m3). (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct capacity |

Question 20 (6 marks)

Given that and , evaluate in each of the following cases:

(a) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses product rule  ü correct value |

(b) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses chain rule  ü correct value |

(c) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses chain rule  ü correct value |

Question 21 (8 marks)

When a byte of data is sent through a network in binary form (a sequence of bits - 's and 's), there is a chance of bit errors that corrupt the byte, i.e. a becomes a and vice versa.

Suppose a byte consists of a sequence of bits and for a particular network, the chance of a bit error is

(a) Determine the probability that a byte is transmitted without corruption, rounding your answer to decimal places. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates binomial distribution  ü indicates probability to calculate  ü correct probability, to dp |

(b) Determine the probability that during the transmission of bytes, at least one of the bytes becomes corrupted. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates correct method  ü correct probability |

A Hamming code converts a byte of bits into a byte of bits for transmission, with the advantage that if just one bit error occurs during transmission, it can be detected and corrected.

(c) Determine the probability that during the transmission of bytes using Hamming codes, at least one of the bytes becomes permanently corrupted. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ü states distribution of failures of a bit byte  ✓ probability that single Hamming code byte corrupted  ü correct probability |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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